"Subspace Matryoshkas" for Multiuser mmWave Communications

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\[ \text{TUM} \]

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From 4G to 5G

1000-fold increase in capacity of cellular networks
Massive MIMO

- large number of antennas at base station
- large antenna gains
- beamforming: send and receive signals through narrow directed beams
- can increase spectral efficiency

Diagram:
- higher spectral efficiency
- greater amount of usable spectrum
- higher cell density
mmWave Massive MIMO

- mmWave frequency bands
  - 30 - 300 GHz
  - underutilized
  - problem: high free space omnidirectional path loss and signal attenuation

- massive MIMO
  - large antenna gains can compensate for high path loss and signal attenuation
  - makes mmWave communication viable
  - can increase amount of usable spectrum

- greater amount of usable spectrum
- higher spectral efficiency
- higher cell density
Advantages and Disadvantages

- mmWave massive MIMO
  - can increase spectral efficiency and amount of usable spectrum
    - large number of antennas increases complexity, power consumption and costs
- complexity, power consumption and costs can be reduced by
  - simplifying RF chains (e.g., by coarse quantization)
  - reducing number of RF chains by moving part of processing from digital into analog domain
System Model
Downlink of Single-Cell Scenario

Base Station (BS)

Mobile Stations (MSs)
Narrowband Block-Fading Channel Model

- vector of transmitted symbols $\mathbf{x} \in \mathbb{C}^{N_{BS}}$
- vector of received symbols $\mathbf{y}_k \in \mathbb{C}^{N_{MS}}$
- channel matrix $\mathbf{H}_k \in \mathbb{C}^{N_{MS} \times N_{BS}}$
- noise vector $\mathbf{\eta}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{N_{MS}})$
Limited Scattering of mm-Wave Channels

small number of scatterers due to high path loss and signal attenuation
→ only a few paths $\ell = 1, 2, \ldots, L_k$


Geometric Channel Model

\[ H_k = \sqrt{\frac{N_{BS} N_{MS}}{L_k}} \sum_{\ell=1}^{L_k} \alpha_{k,\ell} a_{MS}(\phi_{k,\ell}^{MS}) a_{BS}(\phi_{k,\ell}^{BS}) \]

- path gain \( \alpha_{k,\ell} \sim \mathcal{CN}(0, 1) \)
- \( a_{BS}(\phi_{k,\ell}^{BS}) \in \mathbb{C}^{N_{BS}}, a_{MS}(\phi_{k,\ell}^{MS}) \in \mathbb{C}^{N_{MS}} \) array response vectors
2D Geometric Channel Model

array response vectors:
\[ a_{BS} (\phi_{k,\ell}^{BS}, \theta_{k,\ell}^{BS}), \quad a_{MS} (\phi_{k,\ell}^{MS}, \theta_{k,\ell}^{MS}) \]

- \( \phi_{k,\ell}^{BS}, \theta_{k,\ell}^{BS} \) azimuth and elevation angle of departure
- \( \phi_{k,\ell}^{MS}, \theta_{k,\ell}^{MS} \) azimuth and elevation angle of arrival

\[ H_k = \sqrt{N_{BS}N_{MS}} \sum_{\ell=1}^{L_k} \alpha_{k,\ell} a_{MS} (\phi_{k,\ell}^{MS}, \theta_{k,\ell}^{MS}) a_{BS}^{H} (\phi_{k,\ell}^{BS}, \theta_{k,\ell}^{BS}) \]
System Model

- $s_k \sim \mathcal{CN}(0, I_{d_k})$ signal vector for MS $k$ receiving $d_k$ data streams
- $P_k \in \mathbb{C}^{N_{BS} \times d_k}$ precoder for MS $k$:
  \[ \sum_{k=1}^{K} \text{tr} \left( P_k P_k^H \right) \leq P, \quad P \text{ total average transmit power} \]
- $G_k \in \mathbb{C}^{N_{MS} \times d_k}$ equalizer of MS $k$
- $\hat{s}_k \in \mathbb{C}^{d_k}$ processed received signal vector of MS $k$ for $s_k$
Hybrid Precoding

\[ \mathbf{P}_{D,k} \in \mathbb{C}^{N_{RF} \times d_k} \]

\[ d = \sum_{k=1}^{K} d_k \leq N_{RF} \]

hybrid precoder: \( \mathbf{P}_k = \mathbf{P}_A \mathbf{P}_{D,k} \in \mathbb{C}^{N_{BS} \times d_k} \)

\[ \mathbf{x} = \mathbf{P}_A \sum_{k=1}^{K} \mathbf{P}_{D,k} \mathbf{s}_k = \sum_{k=1}^{K} \mathbf{P}_k \mathbf{s}_k \]

BS antennas \( \mathbf{x} \in \mathbb{C}^{N_{BS}} \)
Sum Rate Maximization
Sum Rate Maximization

\[
\begin{align*}
\max \limits_{\{P_k, G_k\}_{k=1}^K} & \sum_{k=1}^K \log_2 \left( \det \left( G_k^H G_k + G_k^H H_k \sum_{j=1}^K P_j P_j^H H_k^H G_k \right) \right) \\
\text{s.t.} & \sum_{k=1}^K \text{tr} \left( P_k P_k^H \right) \leq P, \\
& P_k = P_A P_{D,k} \quad \forall k, \\
& P_A \in \left\{ p \in \mathbb{C} : |p| = \frac{1}{\sqrt{N_{BS}}} \right\}^{N_{BS} \times N_{RF}}, \\
& P_{D,k} \in \mathbb{C}^{N_{RF} \times d_k} \quad \forall k, \\
& G_k \in \mathbb{C}^{N_{MS} \times d_k} \quad \forall k
\end{align*}
\]
State of the Art: Two-Stage Multi-User Hybrid Precoders Algorithm\(^2\) (2SMUHPA)

- 1 data stream per MS/user such that \( N_{RF} = K = d \)
  \( \rightarrow \) precoders/equalizers are vectors: \( P_k = p_k = P_A p_{D,k}, \ G_k = g_k \)
- \( g_k \) implemented in analog domain

\[
\hat{s}_k = g_k^H H_k \sum_{j=1}^{K} p_j s_j + g_k^H \eta_k = g_k^H H_k P_A \sum_{j=1}^{K} p_{D,j} s_j + g_k^H \eta_k 
\]

Stage 1: Maximization of Desired Signal Power for Each User

\[
\{ g_k, p_{A,k} \} = \arg\max_{g \in \mathcal{G}, p_A \in \mathcal{P}_A} \left| g^H H_k p_A \right|^2 \quad \forall k
\]

\[ P_A = \begin{bmatrix} p_{A,1} & p_{A,2} & \cdots & p_{A,K} \end{bmatrix} \]

- \( \mathcal{G}, \mathcal{P}_A \) sets of vectors with constant-modulus entries
- \( H_k = \sqrt{\frac{N_{BS}N_{MS}}{L_k}} \sum_{\ell=1}^{L_k} \alpha_{k,\ell} a_{MS} \left( \phi_{k,\ell}^{MS}, \theta_{k,\ell}^{MS} \right) a_{BS}^H \left( \phi_{k,\ell}^{BS}, \theta_{k,\ell}^{BS} \right) \)
- conjecture: \( p_{A,k} \) and \( g_k \) chosen to be \( a_{BS} \left( \phi_{k,\ell}^{BS}, \theta_{k,\ell}^{BS} \right) \) and \( a_{MS} \left( \phi_{k,\ell}^{MS}, \theta_{k,\ell}^{MS} \right) \) corresponding to \( \alpha_{k,\ell} \) with largest absolute value
  \( \rightarrow g_k \in \mathcal{G}, p_{A,k} \in \mathcal{P}_A \) automatically fulfilled
- \( \hat{s}_k = g_k^H H_k P_A p_{D,k} s_k + \sum_{j=1}^{K} g_k^H H_k p_{A,j} s_j + g_k^H \eta_k \)
  \( j=1 \)
  \( j \neq k \)

interference neglected \( \rightarrow p_{D,j} \in \text{null} \{ g_k^H H_k P_A \}, \ k \neq j \)
Stage 2: Suppression of Multi-User Interference

\[
\hat{H} = \begin{bmatrix}
g_1^H H_1 P_A \\
g_2^H H_2 P_A \\
\vdots \\
g_K^H H_K P_A \\
\end{bmatrix}
\]

\[
P_D = \begin{bmatrix} p_{D,1} & p_{D,2} & \cdots & p_{D,K} \end{bmatrix} = \hat{H}^{-1} \Lambda \Gamma^{\frac{1}{2}}
\]

\[
p_k = P_A p_{D,k} \quad \forall k
\]

- diagonal matrix \( \Lambda \) normalizes columns of \( P_A \hat{H}^{-1} \) to unit norm
- diagonal power loading matrix \( \Gamma \)
  - original version: \( \Gamma = \frac{P}{K} I_K \) gives equal power to all users/streams
  - our extension: optimal power allocation by water-filling

\[
\rightarrow \hat{H} P_D = \begin{bmatrix}
g_1^H H_1 P_A p_{D,1} & g_1^H H_1 P_A p_{D,2} & \cdots & g_1^H H_1 P_A p_{D,K} \\
g_2^H H_2 P_A p_{D,1} & g_2^H H_2 P_A p_{D,2} & \cdots & g_2^H H_2 P_A p_{D,K} \\
\vdots & \vdots & \ddots & \vdots \\
g_K^H H_K P_A p_{D,1} & g_K^H H_K P_A p_{D,2} & \cdots & g_K^H H_K P_A p_{D,K} \\
\end{bmatrix}
\]

\[
= \Lambda \Gamma^{\frac{1}{2}} \text{ such that } p_{D,j} \in \text{null } \{ g_k^H H_k P_A \}, \ k \neq j
\]
Stage 2: Suppression of Multi-User Interference

\[
\hat{H} = \begin{bmatrix}
g_1^H H_1 P_A \\
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\vdots \\
g_K^H H_K P_A \\
\end{bmatrix}
\]

\[
P_D = \begin{bmatrix}
p_{D,1} \\
p_{D,2} \\
\vdots \\
p_{D,K} \\
\end{bmatrix} = \hat{H}^{-1} \Lambda \Gamma^{\frac{1}{2}}
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p_k = P_A p_{D,k}, \quad \forall k
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  - original version: \( \Gamma = \frac{P}{K} I_K \) gives equal power to all users/streams
  - our extension: optimal power allocation by water-filling

\[
\rightarrow \hat{H} P_D = \begin{bmatrix}
g_1^H H_1 P_A p_{D,1} & 0 & \cdots & 0 \\
0 & g_2^H H_2 P_A p_{D,2} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & g_K^H H_K P_A p_{D,K} \\
\end{bmatrix}
\]

\[
= \Lambda \Gamma^{\frac{1}{2}} \text{ such that } p_{D,j} \in \text{null } \{g_k^H H_k P_A\}, \quad k \neq j
\]
LISA — Subspace Matryoshkas
Linear Successive Allocation\(^3\) (LISA)

- developed for fully digital precoding
- successively allocates data streams to users and determines precoders and equalizers for them
  - circumvents the high computational complexity of direct sum rate maximization
  - avoids inherent exhaustive search required for optimal data stream allocation if \(K N_{\text{MS}} > N_{\text{RF}}\)

- 2 steps of interference suppression perfectly match the hybrid architecture

---

estimate for symbol $t_i$ of $i^{th}$ data stream allocated to MS $\pi(i)$:

$$\hat{t}_i = g_i^H H_{\pi(i)} \left( p_i t_i + \sum_{j=1}^{d} p_j t_j \right) + g_i^H \eta_{\pi(i)}$$

- $t_i$ and $\hat{t}_i$ element of $s_{\pi(i)}$ and $\hat{s}_{\pi(i)}$, $i \in \{1, 2, \ldots, d\}$
- $g_i$ and $p_i$ columns of $G_{\pi(i)}$ and $P_{\pi(i)}$
System Model for LISA (2)

\[
\hat{t}_i = g_i^H H_{\pi(i)} p_i t_i + \sum_{\substack{j=1 \\ j \neq i}}^{d} g_i^H H_{\pi(i)} p_j t_j + g_i^H \eta_{\pi(i)}
\]

interference suppressed by zero-forcing (ZF):
\[p_j \in \text{null} \{g_i^H H_{\pi(i)}\}, \ i \neq j\]
Successive Allocation—Enfolding the Matryoshkas

\[ \{ \pi(i), g_i, q_i \} = \arg\max_{k,g,q} |g^H H_k T_i q| \quad \text{s.t.} \quad \|g\|_2 = \|q\|_2 = 1 \]

- user/MS \( \pi(i) \), equalizer \( g_i \) and precoder \( q_i \) for \( i^{th} \) data stream chosen such that gain of its scalar sub-channel within \(\text{null} \{ g_j^H H_{\pi(j)} \}_{j=1}^{i-1} \) is maximum
- \( \pi(i) \): MS \( k \) with largest maximum singular value of \( H_k T_i \)
- \( g_i, q_i \): corresponding left and right singular vector
- \( q_i \in \text{null} \{ g_j^H H_{\pi(j)} \}_{j=1}^{i-1} \)
  - \( Q_i = [q_1 \quad q_2 \ldots \quad q_i] \) such that \( H_{\text{comp},i} Q_i = L_i \)
  - recursive computation of projector \( T_i = I_{NBS} - Q_{i-1} Q_i^H \):
    \[ T_1 = I_{NBS} \]
    \[ T_{i+1} = T_i - q_i q_i^H, \quad i = 1, 2, \ldots, d - 1 \]
- \( P_{\text{eff},d} = Q_d L_d^{-1} \Lambda_d \Gamma_d^{1/2} \rightarrow p_i \in \text{null} \{ g_j^H H_{\pi(j)} \}, j \neq i \)
Subspace Matryoshkas

\[ \mathbb{C}^{N_{\text{BS}}} \supset \text{null} \left\{ g_1^H H_{\pi(1)} \right\} \supset \text{null} \left\{ g_i^H H_{\pi(i)} \right\}_{i=1}^2 \supset \text{null} \left\{ g_i^H H_{\pi(i)} \right\}_{i=1}^3 \supset \cdots \]
Alternative Interpretation—Composite Channel Matrix and Effective Precoder

- composite channel matrix after the $i$th allocation steps:

$$
H_{\text{comp},i} = \begin{bmatrix}
    g_1^\text{H} H^{(1)} \\
    g_2^\text{H} H^{(2)} \\
    \vdots \\
    g_i^\text{H} H^{(i)}
\end{bmatrix} \in \mathbb{C}^{i \times N_{\text{BS}}}
$$

- LQ-decomposition of composite channel matrix with lower triangular matrix $L_i$ and $Q_i^\text{H} Q_i = I_i$:

$$
H_{\text{comp},i} = L_i Q_i^\text{H}
$$

- projector onto null $\{g_j^\text{H} H^{(j)}\}_{j=1}^{i-1}$:

$$
T_i = I_{N_{\text{BS}}} - Q_{i-1} Q_{i-1}^\text{H}
$$

- effective precoder:

$$
P_{\text{eff},i} = \begin{bmatrix}
    p_1 \\
    p_2 \\
    \vdots \\
    p_i
\end{bmatrix} \in \mathbb{C}^{N_{\text{BS}} \times i}
$$
Interference Suppression by ZF in 2 Steps

Step 1:
suppression of interference from successively allocated data streams
\[ H_{\text{comp},d} Q_d = L_d \]
lower triangular matrix

Step 2:
suppression of remaining interference
\[ L_d L_d^{-1} \Lambda_d \Gamma_d^{1/2} = \Lambda_d \Gamma_d^{1/2} \]
diagonal matrix
with diagonal matrix \( \Lambda_d \) such that \( Q_d L_d^{-1} \Lambda_d \) has unit-norm columns
and diagonal power loading matrix \( \Gamma_d \)

\[ \mathbf{P}_{\text{eff},d} = Q_d L_d^{-1} \Lambda_d \Gamma_d^{1/2} \]

\[ H_{\text{comp},d} \mathbf{P}_{\text{eff},d} = \left[ \begin{array}{cccc} g_1^H H_{\pi(1)} p_1 & g_1^H H_{\pi(1)} p_2 & \cdots & g_1^H H_{\pi(1)} p_d \\ g_2^H H_{\pi(2)} p_1 & g_2^H H_{\pi(2)} p_2 & \cdots & g_2^H H_{\pi(2)} p_d \\ \vdots & \vdots & \ddots & \vdots \\ g_d^H H_{\pi(d)} p_1 & g_d^H H_{\pi(d)} p_2 & \cdots & g_d^H H_{\pi(d)} p_d \end{array} \right] \]

\[ = \Lambda_d \Gamma_d^{1/2} \] diagonal matrix such that \( p_j \in \text{null} \{ g_i^H H_{\pi(i)} \}, i \neq j \)
Interference Suppression by ZF in 2 Steps

Step 1:
suppression of interference from successively allocated data streams
\[ H_{\text{comp},d} Q_d \coloneq L_d \] lower triangular matrix

Step 2:
suppression of remaining interference
\[ L_d L_d^{-1} \Lambda_d \Gamma_d^{1/2} \coloneq \Lambda_d \Gamma_d^{1/2} \] diagonal matrix with diagonal matrix \( \Lambda_d \) such that \( Q_d L_d^{-1} \Lambda_d \) has unit-norm columns and diagonal power loading matrix \( \Gamma_d \)

\[ P_{\text{eff},d} = Q_d L_d^{-1} \Lambda_d \Gamma_d^{1/2} \]

\[ H_{\text{comp},d} P_{\text{eff},d} = \begin{bmatrix} g_1^H H_{\pi(1)} p_1 & 0 & \cdots & 0 \\ 0 & g_2^H H_{\pi(2)} p_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & g_d^H H_{\pi(d)} p_d \end{bmatrix} = \Lambda_d \Gamma_d^{1/2} \] diagonal matrix such that \( p_j \in \text{null} \{ g_i^H H_{\pi(i)} \}, \ i \neq j \)
LISA for Hybrid Precoding (LISA Hybrid)

### LISA
- **Step 1:**
  - Suppression of interference from successively allocated data streams by $Q_d$

$$[P_A]_{m,n} = \frac{1}{\sqrt{N_{BS}}} \exp\left(j \arg\left([Q_d]_{m,n}\right)\right)$$

### LISA Hybrid
- **Step 2:**
  - Suppression of remaining interference by $L_d^{-1} \Lambda_d \Gamma_d^{\frac{1}{2}}$

$$P_{\text{eff},d} = Q_d L_d^{-1} \Lambda_d \Gamma_d^{\frac{1}{2}}$$

$$P_{\text{eff},d} = P_A P_D$$

$$\rightarrow H_{\text{comp},d} P_{\text{eff},d} = \Lambda_d \Gamma_d^{\frac{1}{2}}$$

$P_A$ has unit-norm columns.
Large System Analysis Results (1)

- normalization: $\frac{\text{Rate}}{N_{BS}}$
- 1000 channel realizations
- $\alpha = \frac{N_{BS}}{K N_{MS}} = 2$, SNR = 10 dB
- circularly symmetric complex Gaussian channel with $\sigma_H^2 = 1$

 taken from:
Dissertation.
Technische Universität München, 2012
Large System Analysis Results (2)

\[ \alpha = \frac{N_{BS}}{KN_{MS}} = \frac{1}{2} \]

taken from:
Dissertation.
Technische Universität München, 2012
Large System Analysis Results (3)

\[ \alpha = \frac{N_{BS}}{K N_{MS}} \]

\[ N_{BS} = 15 \]

taken from:
Guthy, C.; Utschick, W.; Honig, M. L.: Large system analysis of sum capacity in the Gaussian MIMO broadcast channel. – In: IEEE Journal on Selected Areas of Communications 31 (2, February 2013), pp. 149–159
Numerical Results (1)
Simulation Setup

- comparison of our approach for hybrid precoding (LISA Hybrid) with state of the art (2SMUHPA)

- results obtained in 1000 Monte Carlo runs

- geometric channel model with $L_k = L$ paths per MS:
  - $K$ MSs with an $N_{MS}$-element uniform linear array (ULA) or uniform planar array (UPA) each
  - BS with $N_{BS}$-element UPA and $N_{RF} = K$ RF chains $\rightarrow d \leq K$ data streams
  - angles of arrival and departure selected uniformly at random

- $\text{SNR} = P$
Average Sum Rate vs. SNR for $N_{BS} = 64 (8 \times 8)$, $K = 8$, $L = 1$

- **Capacity**
- **2SMUHPA**
- **2SMUHPA with Water-Filling**
- **LISA**
- **LISA Hybrid**

**Observations:**
- **no significant performance gap** between LISA and LISA Hybrid
- **4 bits (gain: 21%)** for $N_{MS} = 1$
Average Sum Rate vs. SNR for $N_{BS} = 64 (8 \times 8), K = 8, L = 1$

- **Capacity**
- **2SMUHPA**
- **2SMUHPA with Water-Filling**
- **LISA**
- **LISA Hybrid**

**4 bits (gain: 17%)**

**4 bits (gain: 21%)**

No significant performance gap between LISA and LISA Hybrid.
Average Sum Rate vs. SNR for $N_{BS} = 64$, $K = 8$, $N_{MS} = 1$, $L = 3$

- **Capacity**
- **2SMUHPA**
- **2SMUHPA with Water-Filling**
- **LISA**
- **LISA Hybrid**
- **Analog-Only Beamsteering**

For large SNR, 2SMUHPA better than Analog-Only Beamsteering, additional large gain by LISA

9 bits (gain: 72%)
Channel Gains for $N_{BS} = 64$, $K = 8$, $N_{MS} = 1$, $L = 3$, $SNR = 0 \text{ dB}$

LISA Hybrid largely preserves large channel gains of LISA
Average Sum Rate vs. SNR for $N_{BS} = 64$, $K = 8$, $N_{MS} = 2$, $L = 3$

- **Capacity**
- **2SMUHPA**
- **2SMUHPA with Water-Filling**
- **LISA**
- **LISA Hybrid**

$0$ bits (gain: $72\%$)

$N_{MS} = 1$
Average Sum Rate vs. SNR for $N_{BS} = 64$, $K = 8$, $N_{MS} = 2$, $L = 3$

At high SNR, the gap between capacity and LISA is larger since $d \leq N_{RF} = 8 < KN_{MS} = 16$

9 bits (gain: 49%)

$N_{MS} = 2$
Average Sum Rate vs. SNR for $N_{BS} = 64$, $K = 8$, $N_{MS} = 16 (4 \times 4)$, $L = 3$

At high SNR, the gap between capacity and LISA is larger since $d \leq N_{RF} = 8 < N_{BS} = 64$

10 bits (gain: 26%)

SNR [dB]

Average Sum Rate [bits/channel use]

- Capacity
- 2SMUHPA
- 2SMUHPA with Water-Filling
- LISA
- LISA Hybrid
Low-Complexity Version of H-LISA
Reformulation of the LISA Optimization Problem

\[
\{\pi(i), g_i, q_i\} = \arg\max_{k, g, q} |g^H S_{k,i} H_k T_i q| \quad \text{s.t.} \quad \|g\|_2 = \|q\|_2 = 1
\]

- added constraint incorporated into objective by using \( g = S_{k,i} g \) for any vector \( g \in \text{null} \{g^H_j : j = 1, 2, \ldots, i - 1 \land \pi(j) = k\} \)

- \( S_{k,i} \) projector onto \( \text{null} \{g^H_j : j = 1, 2, \ldots, i - 1 \land \pi(j) = k\} \)
  - \( S_{k,1} = I_{N_{\text{MS}}} \)
  - \( S_{k,i+1} = \begin{cases} S_{k,i} - g_i g_i^H, & k = \pi(i) \\ S_{k,i}, & k \neq \pi(i) \end{cases}, \quad i = 1, 2, \ldots, d - 1 \)
Special Structure of Matrix in Objective

plug in expression

\[ H_k = \sqrt{\frac{N_{BS} N_{MS}}{L_k}} \sum_{\ell=1}^{L_k} \alpha_{k,\ell} a_{MS} (\phi_{k,\ell}^{MS}, \theta_{k,\ell}^{MS}) a_{BS}^H (\phi_{k,\ell}^{BS}, \theta_{k,\ell}^{BS}) : \]

\[ S_{k,i} H_k T_i = \sum_{\ell=1}^{L_k} \alpha_{k,\ell} \sqrt{\frac{N_{BS} N_{MS}}{L_k}} S_{k,i} a_{MS} (\phi_{k,\ell}^{MS}, \theta_{k,\ell}^{MS}) (T_i a_{BS} (\phi_{k,\ell}^{BS}, \theta_{k,\ell}^{BS}))^H \]

\[ = \sum_{\ell=1}^{L_k} \alpha_{k,\ell} \sqrt{\frac{N_{BS} N_{MS}}{L_k}} \left\| S_{k,i} a_{MS} (\phi_{k,\ell}^{MS}, \theta_{k,\ell}^{MS}) \right\|_2 \left\| T_i a_{BS} (\phi_{k,\ell}^{BS}, \theta_{k,\ell}^{BS}) \right\|_2 \]

\[ \cdot \frac{S_{k,i} a_{MS} (\phi_{k,\ell}^{MS}, \theta_{k,\ell}^{MS})}{\left\| S_{k,i} a_{MS} (\phi_{k,\ell}^{MS}, \theta_{k,\ell}^{MS}) \right\|_2} \left( \frac{T_i a_{BS} (\phi_{k,\ell}^{BS}, \theta_{k,\ell}^{BS})}{\left\| T_i a_{BS} (\phi_{k,\ell}^{BS}, \theta_{k,\ell}^{BS}) \right\|_2} \right)^H \]
Special Structure of Matrix in Objective

plug in expression

\[ H_k = \sqrt{\frac{N_{BS} N_{MS}}{L_k}} \sum_{\ell=1}^{L_k} \alpha_{k,\ell} a_{MS} (\phi_{k,\ell}^{MS}, \theta_{k,\ell}^{MS}) a_{BS}^H (\phi_{k,\ell}^{BS}, \theta_{k,\ell}^{BS}) : \]

\[ S_{k,i} H_k^T i = \sum_{\ell=1}^{L_k} \alpha_{k,\ell} \sqrt{\frac{N_{BS} N_{MS}}{L_k}} S_{k,i} a_{MS} (\phi_{k,\ell}^{MS}, \theta_{k,\ell}^{MS}) (T_i a_{BS} (\phi_{k,\ell}^{BS}, \theta_{k,\ell}^{BS}))^H \]

\[ = \sum_{\ell=1}^{L_k} \alpha_{k,\ell} \sqrt{\frac{N_{BS} N_{MS}}{L_k}} \| S_{k,i} a_{MS} (\phi_{k,\ell}^{MS}, \theta_{k,\ell}^{MS}) \|_2 \| T_i a_{BS} (\phi_{k,\ell}^{BS}, \theta_{k,\ell}^{BS}) \|_2 \]

\[ \frac{S_{k,i} a_{MS} (\phi_{k,\ell}^{MS}, \theta_{k,\ell}^{MS})}{\| S_{k,i} a_{MS} (\phi_{k,\ell}^{MS}, \theta_{k,\ell}^{MS}) \|_2} = \left( \frac{T_i a_{BS} (\phi_{k,\ell}^{BS}, \theta_{k,\ell}^{BS})}{\| T_i a_{BS} (\phi_{k,\ell}^{BS}, \theta_{k,\ell}^{BS}) \|_2} \right)^H \]
Special Structure of Matrix in Objective

plug in expression

\[
H_k = \sqrt{\frac{N_{BS} N_{MS}}{L_k}} \sum_{\ell=1}^{L_k} \alpha_{k,\ell} a_{MS} (\phi_{k,\ell}, \theta_{k,\ell}) a_{BS} (\phi_{k,\ell}, \theta_{k,\ell})^H.
\]

\[
S_{k,i} H_k T_i = \sum_{\ell=1}^{L_k} \alpha_{k,\ell} \sqrt{\frac{N_{BS} N_{MS}}{L_k}} S_{k,i} a_{MS} (\phi_{k,\ell}, \theta_{k,\ell}) (T_i a_{BS} (\phi_{k,\ell}, \theta_{k,\ell}))^H
\]

\[
= \sum_{\ell=1}^{L_k} \alpha_{k,\ell} \sqrt{\frac{N_{BS} N_{MS}}{L_k}} \left\| S_{k,i} a_{MS} (\phi_{k,\ell}, \theta_{k,\ell}) \right\|_2 \left\| T_i a_{BS} (\phi_{k,\ell}, \theta_{k,\ell}) \right\|_2
\]

\[
\alpha_{k,\ell,i}
\]

\[
S_{k,i} a_{MS} (\phi_{k,\ell}, \theta_{k,\ell}) \left\| S_{k,i} a_{MS} (\phi_{k,\ell}, \theta_{k,\ell}) \right\|_2^2
\]

\[
T_i a_{BS} (\phi_{k,\ell}, \theta_{k,\ell}) \left\| T_i a_{BS} (\phi_{k,\ell}, \theta_{k,\ell}) \right\|_2^2
\]

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Determination of MS, Equalizer and Precoder for $i^{th}$ Data Stream without SVD

- avoid computing SVDs of $S_{k,i} H_k T_i$ for solving

$$\{\pi(i), g_i, q_i\} = \arg \max_{k, g, q} |g^H S_{k,i} H_k T_i q| \quad \text{s.t.} \quad \|g\|_2 = \|q\|_2 = 1$$

- try to solve it by exploiting $S_{k,i} H_k T_i = \sum_{\ell=1}^{L_k} \alpha_{k,\ell,i} a_{MS,k,\ell,i} a_{BS,k,\ell,i}^H$:

  - $\{\pi(i), \ell(i)\} = \arg \max_{k, \ell} |\alpha_{k,\ell,i}|$
  - $g_i = a_{MS,\pi(i),\ell(i),i}$
  - $q_i = \frac{T_i H_{\pi(i)}^H g_i}{\|T_i H_{\pi(i)}^H g_i\|_2}$

null $\{g_{2}^H H_{\pi(2)}\}$
null $\{g_{1}^H H_{\pi(1)}\}$
## Computational Complexity

<table>
<thead>
<tr>
<th>LISA (Hybrid)</th>
<th>( O \left( KLN_{\text{BS}}N_{\text{MS}} + KdN_{\text{BS}}N_{\text{MS}}^2 + dN_{\text{BS}}^2 \right) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-Complexity LISA (Hybrid)</td>
<td>( O \left( KLd \left( N_{\text{BS}} + N_{\text{MS}} \right) \right) )</td>
</tr>
</tbody>
</table>

- computational complexity of LISA (Hybrid) dominated by computing
  - \( K \) channel matrices
    \[
    H_k = \sqrt{\frac{N_{\text{BS}}N_{\text{MS}}}{L}} \sum_{\ell=1}^{L} \alpha_{k,\ell} a_{\text{MS}}(\phi_{k,\ell}, \theta_{k,\ell}) a_{\text{BS}}^H(\phi_{k,\ell}, \theta_{k,\ell})
    \]
  - SVDs of \( K \) matrices \( H_k T_i \) for each of \( d \) data stream allocations
  - projector updates for each of \( d \) data stream allocations
- computational complexity of low-complexity version dominated by computation of weights \( \alpha_{k,\ell,i} \) for all \( K \) MSs, \( L \) paths between BS and each MS, and \( d \) data stream allocations
- for mm-wave communications:
  - \( L \) typically small for mm-wave channels
  - \( N_{\text{BS}}, N_{\text{MS}} \) might be large to overcome high path loss
  
  \( \rightarrow \) computational complexity of low-complexity version much smaller
Numerical Results (2)
Average Sum Rate vs. SNR for $N_{MS} = 1, L = 1$

4 bits (gain: 22%)
Average Sum Rate vs. SNR for $N_{\text{MS}} = 1$, $L = 1$

- Capacity
- 2SMUHPA
- 2SMUHPA with Water-Filling
- LISA
- LISA Hybrid
- Low-Complexity LISA
- Low-Complexity LISA Hybrid

The graph shows the average sum rate in bits per channel use as a function of SNR in dB. The curves indicate that there is no difference between the original and low-complexity version of LISA (Hybrid). Additionally, there is a gain of 22% with 4 bits.

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Average Sum Rate vs. SNR for $N_M = 1$, $L = 3$

- Capacity
- 2SMUHPA
- LISA
- LISA Hybrid

- 9 bits (gain: 72%)

SNR [dB]

Average Sum Rate [bits/channel use]
Average Sum Rate vs. SNR for $N_{MS} = 1$, $L = 3$

A low-complexity version of LISA (Hybrid) only slightly worse than original one

9 bits (gain: 72%)
Average Sum Rate vs. SNR for $N_{\text{MS}} = 16 \ (4 \times 4)$, $L = 3$

The graph shows the average sum rate in bits per channel use versus the SNR in dB. The plot compares different transmission methods:

- **Capacity**
- **2SMUHPA**
- **2SMUHPA with Water-Filling**
- **LISA**
- **LISA Hybrid**

A zoomed-in section highlights that 10 bits can be achieved with a gain of 26% at a specific SNR level. The graph illustrates the performance improvement of the hybrid LISA method compared to traditional approaches.
Average Sum Rate vs. SNR for $N_{MS} = 16 (4 \times 4)$, $L = 3$

- Capacity
- 2SMUHPA
- 2SMUHPA with Water-Filling
- LISA
- LISA Hybrid
- Low-Complexity LISA
- Low-Complexity LISA Hybrid

10 bits (gain: 26%) 

low-complexity version of LISA (Hybrid) significantly reduces computational complexity while maintaining good performance
Sensitivity to Imperfect CSI
Error Model

- assumption:

\[ H_k = \sqrt{\frac{N_{BS}N_{MS}}{L_k}} \sum_{\ell=1}^{L_k} \alpha_{k,\ell} a_{MS}(\phi_{k,\ell}^{MS}, \theta_{k,\ell}^{MS}) a_{BS}^{H}(\phi_{k,\ell}^{BS}, \theta_{k,\ell}^{BS}) \]

estimated by estimating \( \alpha_{k,\ell}, \phi_{k,\ell}^{X} \) and \( \theta_{k,\ell}^{X}, X \in \{MS, BS\} \)

- estimated complex path gains:

\[ \hat{\alpha}_{k,\ell} = \sqrt{1 - e^2 \alpha_{k,\ell}} + e \beta_{k,\ell}, \beta_{k,\ell} \sim \mathcal{CN}(0, 1), e \in [0, 1] \]

\( \rightarrow \) MSE: \( E \left[ |\hat{\alpha}_{k,\ell} - \alpha_{k,\ell}|^2 \right] = 2 \left( 1 - \sqrt{1 - e^2} \right) \)

- estimated angles:

\[ \hat{\phi}_{k,\ell}^{X} = \phi_{k,\ell}^{X} + \epsilon_{\phi_{k,\ell}^{X}}, \epsilon_{\phi_{k,\ell}^{X}} = \begin{cases} -\Delta \text{ with probability 0.5} \\ \Delta \text{ with probability 0.5} \end{cases} \]

\[ \hat{\theta}_{k,\ell}^{X} = \theta_{k,\ell}^{X} + \epsilon_{\theta_{k,\ell}^{X}}, \epsilon_{\theta_{k,\ell}^{X}} = \begin{cases} -\Delta \text{ with probability 0.5} \\ \Delta \text{ with probability 0.5} \end{cases} \]
Average Sum Rate vs. SNR for $N_{BS} = 64$, $N_{MS} = 16$, $L = 3$

- 2SMUHPA
- LISA
- H-LISA
- Low-C. LISA
- Low-C. H-LISA

Estimation errors lead to significant performance degradation at high SNR.
Average Sum Rate vs. SNR for $N_{\text{BS}} = 64$, $N_{\text{MS}} = 16$, $L = 3$, $e = 0$, H-LISA

Error in angles has severe impact on performance at high SNR.
Average Sum Rate vs. SNR for $N_{BS} = 64$, $N_{MS} = 16$, $L = 3$, $e = 0$, 2SMUHPA

2SMUHPA has similar sensitivity to errors in angles as H-LISA
Average Sum Rate vs. SNR for $N_{BS} = 64$, $N_{MS} = 16$, $L = 3$, $\Delta = 0^\circ$, H-LISA

Average Sum Rate [bits/channel use] vs. SNR [dB]

- $e = 0$
- $e = 0.1$
- $e = 0.2$
- $e = 0.3$
- $e = 0.4$

Error in complex path gains has severe impact on performance at high SNR too.
Average Sum Rate vs. SNR for $N_{BS} = 64$, $N_{MS} = 16$, $L = 3$, $\Delta = 0^\circ$, 2SMUHPA

2SMUHPA has similar sensitivity to errors in complex path gains as H-LISA
Conclusions
Thank You!
References I


Appendix A: Block-Diagonalization
ZF Constraints for Block-Diagonalization\textsuperscript{4} (BD)

\[
\hat{s}_k = G_k^H H_k P_k s_k + \sum_{j=1}^{K} G_k^H H_k P_j s_j + G_k^H \eta_k
\]

multi-user interference suppressed if $H_k P_j = 0$, $k \neq j$

\[ P_k = \tilde{V}_k^{(0)} V_k^{(1)} \Lambda_k^{\frac{1}{2}} \]

- block-diagonalization by orthonormal basis \( \tilde{V}_k^{(0)} \) for null \( \left\{ \begin{bmatrix} H_1^T & \ldots & H_{k-1}^T & H_{k+1}^T & \ldots & H_K^T \end{bmatrix}^T \right\} \):

\[
\begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_K \end{bmatrix} \begin{bmatrix} P_1 & P_2 & \ldots & P_K \end{bmatrix} = \begin{bmatrix} H_1 P_1 & 0 & \ldots & 0 \\ 0 & H_2 P_2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & H_K P_K \end{bmatrix}
\]

\[ \rightarrow H_k P_j = 0, \ k \neq j: \hat{s}_k = G_k^H H_k P_k s_k + G_k^H \eta_k \]

- sum rate maximization under ZF constraints and power constraint:

\[
G_k^H H_k P_k = G_k^H H_k \tilde{V}_k^{(0)} V_k^{(1)} \Lambda_k^{\frac{1}{2}} = \Sigma_k \Lambda_k^{\frac{1}{2}} \text{ diagonalized by } G_k = U_k^{(1)} \text{ and } V_k^{(1)} \text{ from SVD}
\]

\[
H_k \tilde{V}_k^{(0)} = \begin{bmatrix} U_k^{(1)} & U_k^{(0)} \end{bmatrix} \begin{bmatrix} \Sigma_k & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_k^{(1)} \\ V_k^{(0)} \end{bmatrix}^H
\]
\( P_k = \tilde{V}_k^{(0)} V_k^{(1)} \Lambda_k^{\frac{1}{2}} \)

- block-diagonalization by orthonormal basis \( \tilde{V}_k^{(0)} \) for
  \[
  \text{null}\left\{ \begin{bmatrix} H_1^T & \ldots & H_{k-1}^T & H_{k+1}^T & \ldots & H_K^T \end{bmatrix}^T \right\}:
  \]

\[
\begin{bmatrix}
  H_1 \\
  H_2 \\
  \vdots \\
  H_K
\end{bmatrix}
\begin{bmatrix}
P_1 \\
P_2 \\
\vdots \\
P_K
\end{bmatrix}
= 
\begin{bmatrix}
  H_1 P_1 & 0 & \ldots & 0 \\
  0 & H_2 P_2 & \ldots & 0 \\
  \vdots & \vdots & \ddots & \vdots \\
  0 & 0 & \ldots & H_K P_K
\end{bmatrix}
\]

\[ \Rightarrow H_k P_j = 0, \; k \neq j: \hat{s}_k = G_k^H H_k P_k s_k + G_k^H \eta_k \]

- sum rate maximization under ZF constraints and power constraint:
  
  \[ \mathbf{G}_k^H H_k P_k = \mathbf{G}_k^H H_k \tilde{V}_k^{(0)} V_k^{(1)} \Lambda_k^{\frac{1}{2}} = \Sigma_k \Lambda_k^{\frac{1}{2}} \text{ diagonalized by} \]

  \[ \mathbf{G}_k = U_k^{(1)} \text{ and } V_k^{(1)} \text{ from SVD} \]

\[ H_k \tilde{V}_k^{(0)} = [U_k^{(1)} \quad U_k^{(0)}] \begin{bmatrix}
  \Sigma_k & 0 \\
  0 & 0
\end{bmatrix} [V_k^{(1)} \quad V_k^{(0)}]^H \]
\[ P_k = \tilde{V}_k^{(0)} V_k^{(1)} \Lambda_k^{1/2} \]

- block-diagonalization by orthonormal basis \( \tilde{V}_k^{(0)} \) for

\[
\text{null} \left\{ \begin{bmatrix} H_1^T & \ldots & H_{k-1}^T & H_{k+1}^T & \ldots & H_K^T \end{bmatrix}^T \right\}:
\]

\[
\begin{bmatrix}
  H_1 \\
  H_2 \\
  \vdots \\
  H_K \\
\end{bmatrix}
\begin{bmatrix}
  P_1 \\
  P_2 \\
  \vdots \\
  P_K \\
\end{bmatrix} =
\begin{bmatrix}
  H_1 P_1 & 0 & \ldots & 0 \\
  0 & H_2 P_2 & \ldots & 0 \\
  \vdots & \vdots & \ddots & \vdots \\
  0 & 0 & \ldots & H_K P_K \\
\end{bmatrix}
\]

\( \rightarrow H_k P_j = 0, \ k \neq j: \hat{s}_k = G_k^H H_k P_k s_k + G_k^H \eta_k \)

- sum rate maximization under ZF constraints and power constraint:

\[ G_k^H H_k P_k = G_k^H H_k \tilde{V}_k^{(0)} V_k^{(1)} \Lambda_k^{1/2} = \Sigma_k \Lambda_k^{1/2} \text{ diagonalized by} \]

\( G_k = U_k^{(1)} \) and \( V_k^{(1)} \) from SVD

\[ H_k \tilde{V}_k^{(0)} = \begin{bmatrix} U_k^{(1)} & U_k^{(0)} \end{bmatrix} \begin{bmatrix} \Sigma_k & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_k^{(1)} & V_k^{(0)} \end{bmatrix}^T \]
\[ P_k = \tilde{V}_k^{(0)} V_k^{(1)} \Lambda_k^{\frac{1}{2}} \]

- block-diagonalization by orthonormal basis \( \tilde{V}_k^{(0)} \) for

\[
\text{null}\left\{ \begin{bmatrix} H_1^T & \ldots & H_{k-1}^T & H_{k+1}^T & \ldots & H_K^T \end{bmatrix}^T \right\}:
\]

\[
\begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_K \end{bmatrix} \begin{bmatrix} P_1 & P_2 & \ldots & P_K \end{bmatrix} = \begin{bmatrix} H_1 P_1 & 0 & \ldots & 0 \\ 0 & H_2 P_2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & H_K P_K \end{bmatrix}
\]

\[ \implies H_k P_j = 0, \ k \neq j: \hat{s}_k = G_k^H H_k P_k s_k + G_k^H \eta_k \]

- sum rate maximization under ZF constraints and power constraint:

\[ G_k^H H_k P_k = G_k^H H_k \tilde{V}_k^{(0)} V_k^{(1)} \Lambda_k^{\frac{1}{2}} \Lambda_k^{\frac{1}{2}} = \Sigma_k \Lambda_k^{\frac{1}{2}} \text{ diagonalized by } G_k = U_k^{(1)} \text{ and } V_k^{(1)} \text{ from SVD} \]

\[
H_k \tilde{V}_k^{(0)} = \begin{bmatrix} U_k^{(1)} \\ U_k^{(0)} \end{bmatrix} \begin{bmatrix} \Sigma_k & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_k^{(1)} \\ V_k^{(0)} \end{bmatrix}^H
\]
block-diagonalization by orthonormal basis $\tilde{V}_k^{(0)}$ for

$$P_k = \tilde{V}_k^{(0)} \tilde{V}_k^{(1)} \Lambda_k^{\frac{1}{2}}$$

null \(\{H_1^T \ldots H_{k-1}^T H_{k+1}^T \ldots H_K^T\}^T\):

$$\begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_K \end{bmatrix} \begin{bmatrix} P_1 & P_2 & \ldots & P_K \end{bmatrix} = \begin{bmatrix} H_1 P_1 & 0 & \ldots & 0 \\ 0 & H_2 P_2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & H_K P_K \end{bmatrix}$$

$\rightarrow H_k P_j = 0, \ k \neq j$: $\hat{s}_k = G_k^H H_k P_k s_k + G_k^H \eta_k$

sum rate maximization under ZF constraints and power constraint:

- $G_k^H H_k P_k = G_k^H H_k \tilde{V}_k^{(0)} V_k^{(1)} \Lambda_k^{\frac{1}{2}} = \Sigma_k \Lambda_k^{\frac{1}{2}}$ diagonalized by $G_k = U_k^{(1)}$ and $V_k^{(1)}$ from SVD

- $H_k \tilde{V}_k^{(0)} = \begin{bmatrix} U_k^{(1)} & U_k^{(0)} \end{bmatrix} \begin{bmatrix} \Sigma_k & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_k^{(1)} \\ V_k^{(0)} \end{bmatrix}^H$

- diagonal elements of all diagonal power loading matrices $\Lambda_k$ determined by water-filling on diagonal elements of all $\Sigma_k$
BD with Exhaustive Search

- optimal allocation of data streams to MSs combinatorial problem involving exhaustive search
- assumption:
  each MS receives either no or maximum number of data streams (number of antennas $N_{MS}$) → data stream allocation reduces to user selection
- maximum number of MSs that can be simultaneously supported if total number of data stream is limited by number of RF chains $N_{RF}$:
  \[
  \hat{K} = \frac{N_{RF}}{N_{MS}}
  \]
- exhaustive search:
  perform BD for each set of $1 \leq i \leq \hat{K}$ MSs and select BD solution with largest sum rate
- complexity: $\mathcal{O} \left( \sum_{i=1}^{\hat{K}} \binom{K}{i} \left( i^3 N_{BS} N_{MS}^2 + i N_{BS}^2 N_{MS} \right) \right)$

---

Average Sum Rate vs. SNR for $N_{\text{MS}} = 1$, $L = 3$, $\hat{K} = 8$

![Graph showing average sum rate vs. SNR for different algorithms.]

- **Capacity**
- **2SMUHPA**
- **2SMUHPA with Water-Filling**
- **LISA**
- **LISA Hybrid**
- **BD Exhaustive Search**

**BD as good as LISA for $N_{\text{MS}} = 1$**
Average Sum Rate vs. SNR for $N_{MS} = 2$, $L = 3$, $\hat{K} = 4$

[Graph showing average sum rate vs. SNR for different schemes such as Capacity, 2SMUHPA, 2SMUHPA with Water-Filling, LISA, LISA Hybrid, and BD Exhaustive Search. The BD scheme is significantly worse than LISA (Hybrid) for $N_{MS} = 2$.]

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